Please check that this question paper contains 9 questions and 02 printed pages within first ten minutes.

[Total No. of Questions: 09]
Uni. Roll No. ……………

[Total No. of Pages: 02]

MORNING
14 JAN 2023

Program: B.Tech. (Batch 2018 onward)
Semester: 1/2
Name of Subject: Mathematics-I
Subject Code: BSC-103
Paper ID: 15927
Scientific calculator is Not Allowed

Time Allowed: 03 Hours

Max. Marks: 60

NOTE:

1) Parts A and B are compulsory
2) Part-C has Two Questions Q8 and Q9. Both are compulsory, but with internal choice
3) Any missing data may be assumed appropriately

Part – A

[Marks: 02 each]

Q1

a) State Cayley Hamilton Theorem.
b) Evaluate $\lim_{x\to\infty}(1+x)^{1/x}$.
c) Prove that $\frac{1}{D}X=\int X\,dx$ where $D=\frac{dy}{dx}$ and X is a function of x.
d) Give an example of a series which is conditionally convergent but not absolutely convergent.
e) Evaluate the improper integral $\int_0^\infty e^{-2x}x^5\,dx$.
f) Solve the equation $xp^2-yp+a=0$.

Part – B

[Marks: 04 each]

Q2. Expand $\log x$ in powers of $(x-1)$ using Taylor Theorem.

Q3. Using Cauchy Integral test, discuss the convergence of the series $\sum_{n=1}^{\infty}\frac{n}{(n^2+1)^2}$.

Q4. Find the general solution of the differential equation
$(3x^2y^3e^y+y^3+y^2)dx+(x^3y^3e^y-xy)dy=0$.

P.T.O.

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Q5. Prove that $\int_0^{\pi/2} \sin^p\theta \cos^q\theta \, d\theta = \dfrac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{2\,\Gamma\left(\frac{p+q+2}{2}\right)}$. Hence Evaluate $\Gamma\left(\frac{1}{2}\right)$.

Q6. Discusss the consistency of the following system of equations $2x+3y+4z=11$, $x+5y+7z=15$, $3x+11y+13z=25$. If found consistent, solve it.

Q7. Solve by method of variation of parameter $\dfrac{d^2y}{dx^2}+y=\operatorname{cosec} x$.

Part – C **[Marks: 12 each(06 for each subpart if any)]**

Q8. Solve $x^2\dfrac{d^2y}{dx^2}-3x\dfrac{dy}{dx}+5y=\sin(\log x)$.

OR

(i) Solve the differential equation $xy(1+xy^2)\dfrac{dy}{dx}=1$

(ii) Solve $p(p+y)=x(x+y)$.

Q9. Discuss for what values of x does the series $\sum_{n=1}^{\infty}\dfrac{(n!)^2}{(2n)!}x^{2n}$ converge/ diverge?

OR

Find a matrix P which transforms the matrix $\begin{bmatrix}1 & 1 & 3\\ 1 & 5 & 1\\ 3 & 1 & 1\end{bmatrix}$ into a diagonal form.

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